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UNIFYING QUANTITATIVE METHODOLOGY IN SOCIAL RESEARCH

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Abstract

A case is made for representing quantitative methods in use in the social sciences within a unified framework based on structural equation methodology (SEM). Most of the methods now in use are shown in their SEM representation. It is suggested that the visual and verbal representations of SEM are of most use, while specific estimation and hypothesis testing methods play a lesser role. The change to a common verbal and visual representation for quantitative methods allows better understanding of quantitative methods for students, easier communication among researchers, and the basis for new improvements in quantitative approaches to social research.

UNIFYING QUANTITATIVE METHODOLOGY IN SOCIAL RESEARCH

Social sciences have utilized for nearly two centuries quantitative methods that have been loosely covered with the descriptive umbrella called statistics. From a conceptual perspective, however, statistics as a term has almost no meaning. To a layperson it connotes numbers somehow associated with human phenomena. To a social science practitioner, whether educator, social worker, or counselor, it is associated with a fairly unpleasant course or sequence of courses, often confused between measurement and research or simply grouped together, with secondary association to irrelevant seeming research studies in academic journals. To social scientists statistics often is associated with the particular methodological tradition in which they trained as graduate students, such as experimental design, linear regression, or chi square categorical modeling. Even the most sophisticated social researchers and methodologists compartmentalize most of statistics into niches rather than take an integrative view. To some extent these perspectives are warranted, because the approaches taken in methodology and statistics texts, the focus in research training, and the writing in social research reinforce a compartmented approach to statistics, and more generally, to all quantitative methods in social research. It is argued here that there have been sufficient advances in quantitative thinking and theory over the last three decades to propose a unified perspective on much of statistical and quantitative methodology. This has the promise of reconciling much of the disjoint in conceptualization and practice observed today in social research.

While the history of quantitative thought in social research has been reviewed well by many authors (Porter, 1986;Glass & Hopkins, 1996), the perspectives taken in statistical texts generally follow the statistical foci of the disciplines targeted, such as economics,

psychology, sociology, and biology (Willson, 1980). Papers dealing with the use of methods and techniques (Kaplan, 1965) have tended to discuss them in such terms also (Edgington, 1972; Willson, 1980; Goodwin & Goodwin, 1985). Research methods textbooks have focused on key methods such as analysis of variance and regression, but usually treat them as separate methods in use and in conceptualization (eg., Borg, Gall, & Borg, 1994; Kerlinger, 1986; Kerlinger & Pedhazur, 1985; Krathwohl, 1995). Pedhazur and Schmelkin (1994) went further than anyone else in attempting to integrate measurement, statistics, and design considerations, although their formal integration did not occur until their last chapters, and they did not truly present a unified conceptualization throughout their book. Similarly, no current statistics book has made even a token attempt to produce an integrated approach to the range of quantitative methods in use today. Most statistics books are interchangeable in their predictable selection of topics, arrangement, and presentation. They themselves are quite distinct from measurement texts, which typically attempt to combine a modicum of psychometric theory with substantive issues related to the purpose, content, and use of psychological and educational tests. Intermediate and advanced psychometrics texts are distinguished by their rarity, which indicates the infrequency with which psychometric topics are taught in any depth in graduate school. The dissonance produced by such disjointed treatments of these topics is reflected in the often haphazard and situated thinking displayed by social science professionals and researchers with respect to quantitative methodology. To support this strong assertion, one need but ask such a professional to produce a concept map of the topics in social research methodology, statistics, and measurement. Such maps are more likely to follow the independent development the respondents were presented in

graduate courses than a coherent, integrated production. The respondents should not be criticized, for their instructors have likely never attempted the task themselves, nor have the supporting textbooks made a serious effort to support such integrated thinking about the topics.

Unification of quantitative methods as presented here is proposed as a conceptual framework for representation of mathematical models of variables defined and operationalized by social scientists based on Structural Equation Modeling (SEM). While there is a general covering theory for statistical estimation and testing of parameters and models under SEM, it is not necessarily efficient for all applications. Indeed, most statistical techniques have been investigated for their properties and limitations with respect to the assumptions about the distributions of the random variables considered. What is argued here is that the same SEM representation can cover these disparate techniques while allowing the specifics already developed for application in estimation or testing of hypotheses. I will not plunge into the debate over hypothesis testing, since it is not specifically required for the framework presented here. Further, a unified quantitative representation will not cover all aspects of the research process.

A research method is more than the statistical analysis attached to it. Analysis of variance is a statistical technique associated with experimental and quasi-experimental design, yet an exquisitely fine experimental design may be implemented without any statistical treatment by analysis of variance. Further, the experimental design method itself may draw on other methods, such as randomization, random selection, sampling methods, psychometric methods, philosophical analysis, hermeneutics, and text analysis, none of which has anything to do with statistical analysis of variance. Since the focus of

this paper is on quantitative methods, it will not attempt to link such methods or more parochial techniques developed within particular disciplines and within methods. Nevertheless, the primary argument made here is that an integrated quantitative methodology exists which remedies much of the situated thinking inherent in current textbooks, coursework, and research reporting. It is further argued that most of the quantitative methods commonly used in social science fields such as sociology, education, and psychology are representable within the SEM framework, and that the integrated approach advocated here will help greatly to promote common thought, discussion, representation, and understanding among social science practitioners, teachers, and researchers.

FRAMEWORK

Co-relationship is the basis for most conceptual development in social science and for all statistical analysis of two or more variables. This is most frequently formalized as correlation for variables that are conceived as interval or ratio (see Glass & Hopkins, 1996, for a discussion of S. S. Stevens' division of number into nominal, ordinal, interval, and ratio). I will focus on correlation and its unstandardized form, covariance, while recognizing the presence of competing nonparametric statistical methods, and I will suggest that they may be incorporated into a unified quantitative method through summary representations of their information. Various quantitative methods will be represented in successively complex cases, followed by a brief discussion of estimation and hypothesis testing. No claim is made that all SEM representations need to be estimated using SEM procedures. It is the common language, representation, and

organizational taxonomy provided by SEM that will prove most useful in unifying quantitative thinking.

All social science models, whether linear or nonlinear, routinely include error variables. Historically, these have been associated with the outcome or endogenous variable in techniques such as multiple regression and ANOVA. While a typical assumption in traditional formulations is made that the errors are independent and identically distributed, assumptions about error variances and covariances can be replaced with parameters to be estimated in SEM. Further, errors and their variances and covariances may be specified for independent variables in the model, which is not usually the case in GLM analyses. Additionally, nonlinear processes involving error-variable covariations can be represented.

One additional benefit of a unified SEM-based representation is the ease of considering alternative models, both linear and nonlinear. While algebraic models are often difficult for researchers to access conceptually, visual representations can be much more liberating for researchers with limited mathematical proficiency. Indeed, good conceptual thought should dominate quantitative method rather than quantitative "hammers" that limit modeling. While not all nonlinear models can be represented in SEM form, many useful product models can be represented. Several examples will be shown.

TWO OBSERVED VARIABLE MODELS

Pearson correlation has provided for over a century a means to represent covariation of two observed variables in a standardized metric between –1 and +1. The standardization was thought necessary to eliminate the arbitrariness of the scale differences in covariation when we measure variables with inherently different units, but

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unstandardized covariance carries the same information. Only a few decades later Sewell Wright (1918) brilliantly blended the mathematical, analytical representation of covariation with the visual representation of the path model, shown in two forms in Figure 1.

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insert Fig. 1 here)

Figure 1a represents covariation as uninterpreted co-relationship between two variables, X and Y; the curved arrow indicates an inability or unwillingness of the researcher to specify direction of relationship. Figure 1b represents a directed relationship in which X's covariation results in or "causes" Y's covariation exactly, such as is found in deterministic fields such as physics. While the concept and use of cause in social sciences has been extensively debated (Cook & Campbell, 1979), the general consensus of researchers seems to be that cause is a useful concept for discussing theory and experimental evidence; its role in nonexperimental, observational research is as widespread if more debatable. Y may covary without any change in X if an independent error is specified, the representation for simple linear regression, shown as Fig. 1c. In classical regression the error term associated with X is assumed to be zero, while in the more general case error may not have a zero path. While Wright and others since (see Bollen, 1989) have focused on path analysis as a nonexperimental method, experimental models are represented the same way. For example, Fig. 1d can as easily represent a randomized two group t-statistic by labeling X as C (for contrast). Thus, in Fig. 1d all two-group designs can be parsimoniously represented. Instead of a t-statistic the

relationship between group membership C and outcome Y can be represented by the associated Pearson correlation for the two variables. Note that no distributional assumptions have been yet made. It is, however, a small jump to place the asterisk after the path coefficient to denote significance at a given alpha level, and all the appropriate distributional assumptions will be needed. There is no need at this level to distinguish between correlation estimated by Pearson correlation or Spearman rank correlation for the nonparametric analysis with Y rescaled to ranks. The Mann-Whitney U-statistic or Wilcoxon statistic will provide the test for the asterisk instead of Student's t. Even one dichotomous variable C_1 covarying with another dichotomous variable C_2 has meaning, as with political affiliation (party or independent) in relation to gender (male or female). How to calculate an appropriate coefficient to represent the relationship has been the provenance of disciplines that grapple with such data routinely. Glass, McGaw, and Smith (1983) approached this problem in dealing with meta-analysis; their table 5.8 for representing data as correlations can support this effort.

THREE OBSERVED VARIABLE MODELS

Wright provided most of the insight needed to sort out the various relationships among

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insert Fig. 2 here)

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three observed variables, and this is the minimum number needed to understand ANOVA and multiple regression, still among the most widely used statistical methods. Figure 2a shows a classical three-group randomized ANOVA as a two contrast representation, C_1 and C_2. Since the effects are independent, they contribute independently to the outcome

Y and there is no curved arrow between them. Again, if all variables are in standard score form (including the contrasts), the squared path coefficients can be summed to produce eta-squared (η^2), or its more common nomenclature today, squared multiple correlation (R^2). Fig. 2b represents two nonorthogonal contrasts that might be of primary interest to the researcher. Note that the path representation for either adjusts for the covariation with the other. This is identical to the multiple regression representation in Fig. 2c, long used in nonexperimental research. It is usual in multiple regression to interpret nonstandardized coefficients (b-weights) instead of the standardized (beta-weight) coefficients, although Bollen (1989) has correctly pointed out that beta-weights represent either elasticities or standardized regression weights, and interpretation of the magnitudes may be useful in some cases, not in others. For example, the b-weight for contrasts C_1 or C_2 inform the researcher of the effect for the difference between two weighted sets of means. Divided by the standard deviation of Y, the result is the effect size g (see Kirk, 1996, for a discussion of dummy, effect, and contrast coding in terms of b-weights). Of interest is the nonorthogonal case in Fig. 2b. It is not common in statistics texts to formally represent the correlation between pairs of nonorthogonal contrasts, yet in path representation the correlation provides a useful indicator of the relatedness induced for the contrasts of interests.

In Fig. 2d is the truly unique contribution of Wright, the ordered path model. In this model X_1 is exogenous, representing the independent variables of all multiple regression cases discussed above, and Y_1 and Y_2 are endogenous, yet possessing order between themselves, with Y_1 predicting Y_2. Both endogenous variables are predicted or caused by X_1. This allows representation of direct effects (X_1 on Y_2 or Y_1 on Y_2) and indirect

effects X_1 on Y_2 through Y_1. In standardized form the total correlation between X_1 and Y_2 is decomposed into these effects; the direct effect also represents the regression weight for X_1 in predicting Y_2. This effect may be quite misleading in terms of X_1's effect on Y_2, however, since the indirect effect may be substantial or even greater than the direct effect. Bollen (1989) discussed this in his presentation of path modeling. Note that new variables are created in the model through the errors or disturbances for Y_1 and Y_2

The general linear model formulation of ANOVA points to its formal identity with multiple regression, and mixtures of contrast and interval exogenous variables, termed covariates in ANOVA methodology, pose no special problems. Fig. 2e represents the full randomized ANCOVA design with covariate independent of treatment. In Fig. 2f a quasi-experimental situation is shown in which treatment groups are not randomized, and covariate values are different in mean for the groups. Thus, the exogenous covariate is predictive of the treatment, which now becomes endogenous but causally prior to the outcome. The indirect effect of the covariate on outcome is distinguishable from its direct effect.

FACTOR MODELS

One of the major successes of structural equation modeling is the formal union between psychometrics and the general linear model. This mathematical and conceptual success brought traditions each a century old into correspondence. From Hoyt's (1941) representation of reliability theory as a repeated measures design to Cronbach, Rajaratnam, Nanda, and Glaser's (1972) formalization of generalizability theory, researchers were long aware of the connections, but Joreskog's (1973) paper formally

demonstrated how classical measurement theory could be represented in an SEM framework. Fundamental to that representation was the latent variable model.

A single latent variable, or factor, can uniquely be identified mathematically or represented by a minimum of three observed variables, shown in Fig. 3a. If the latent variable is restricted by scaling it to unit variance, in standardized units the paths from latent variable to observed are the factor loadings (structure coefficients). Bollen (1989) discussed the minimum requirements for unique representation of a single factor. Parallel, tau-equivalent, and congeneric models are all cases of this model with differing assumptions about the loadings and error variances.

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insert Fig. 3 here)

Classical reliability follows from the two variable representation via Spearman-Brown step-up of the correlation between the two variables, or from the eigenvalue of the latent variable, equal to the sum of the squared loadings and a linear function of the reliability.

Two latent variables are identified by a minimum of four observed variables, two per latent variable, with no restrictions on the loadings or error variances, pictured in Fig. 3b. Paradoxically, the latent variables must have a nonzero covariance; this is clear since otherwise each latent variable would have only two observed variables to identify it, violating the general case for a single latent variable.

FOUR OBSERVED VARIABLE MODELS

The two latent factor model discussed above (Fig. 3b) fits in this section. The balanced factorial ANOVA is represented in its simplest full model form in Fig. 4a. Each

main effect and the two-way interaction are represented as a contrast in the 2 x 2 design. Fig. 4b shows another four-variable application based on ANOVA, the 2 x 2 design with unbalanced sample size across cells. Since the lack of balance induces covariation among the exogenous contrasts, estimation of the direct effects is equivalent to Type III or unique contributions to the outcome. Fig. 4c illustrates the path model with both indirect and unanalyzed effects. The two exogenous variables, X_1 and X_2 are correlated but no direction of relationship is known or hypothesized between them. Both predict the endogenous variables Y_1 and Y_2 ,which are directionally related.

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insert Fig. 4 here)

From a GLM perspective it can be shown that all GLM models are special cases of canonical correlation (Thompson, 1984). Essentially, the canonical model posits latent factors that are endogenous with respect to one set of variables, the X set, causally prior with respect to the other, Y set. Fan (1997) has shown how this is a particular representation of the SEM model, shown in Fig. 4d. It should be noted that formally canonical correlation constructs two latent variables, one for each set X and Y, that are then correlated. While the representation in Fig. 4e shows this, Fan's representation is the one actually used if one were to perform a canonical analysis using SEM.

MULTIVARIATE MODELS

Even more complex multivariate models are then easily represented by adding additional sets of observed variables in various relationships to the X and Y sets in Fig. 4d. Multivariate analysis of variance (MANOVA) is a special case with the X set

replaced by contrasts, and MANCOVA is represented by mixing the X variable set between contrast and continuous variables, with paths between them dependent on the specifics of the model to be represented. Similarly, predictive discriminant analysis (PDA) is represented by leaving the X variable set as continuous measures and replacing the Y set with contrasts. In fact, this formulation is more focused than ordinary discriminant analysis, since the path coefficients of the X set, structure coefficients in PDA, maximally discriminate the set of contrasts defined by the researcher.

Latent structural modeling allows the inclusion of latent variables at any point in the four variable path model, represented in Fig. 5, which is shows both indirect and unanalyzed relationships. Note that at least eight observed variables are required. Of course, implicit in Fig. 5 are the various possibilities with paths removed between different variables, latent or observed. Also possible are more complex relationships among the observed variables, such as loadings on two latent factors, paths added between observed variables, and paths among errors. This potential for complexity gives the SEM representation its richness and advantage over GLM approaches.

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insert Fig. 5 here)

NONLINEAR MODELS

One of the virtues of the visual representation of SEMs is that complex models with difficult algebraic representations are made more understandable through the drawings. For example, a four variable path model is shown in Fig. 6. The dual arrows between the Y variables suggest a reciprocal influence of the two variables on each other. Such

nonlinear relationship concepts have not been made available to students in classical statistical courses, yet such systems are routinely discussed in social science courses. While there are numerous methodological difficulties in estimating these paths, consideration of such models has occurred and various requirements for their estimation developed (see Bollen, 1989).

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insert Fig. 6 here)

Another nonlinear process is shown in Fig. 7, in which X_{t-1} represents the original time series of data and X_t the same data lagged one time point (also the Y process in the figure). Note that the nonlinearity consists of allowing errors at one time point to affect the observed data at the next time point, in addition to allowing errors to correlate over time (moving average autocorrelation in Y) and observed data to correlate over time in a specific way (autoregression in X). Willson (1995) discussed these models in detail.

Neither of the nonlinear models represented above is discussed in the typical GLM-based statistics course sequence, yet both are meaningful models for social science data. The fact that they can both be represented from the same perspective as ANOVA or regression models suggests a great deal of generality and flexibility from the SEM perspective.

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insert Fig. 7 here)

A TAXONOMY FOR STRUCTURAL EQUATION MODELS

Taxonomies are two-edged weapons; they can help organize information that was for many students ill-structured or fragmented. They can also constrain thinking about a topic and restrict insightful perspectives. Risking this, a web diagram is presented in Fig. 8. The general approach has been to use a pyramidal scheme, with simpler statistical methods lower than more complex models that subsume them.

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insert Fig. 8 here)

Several statistical methods are linked in the diagram that have not been discussed formally here. IRT modeling is represented as a special case of the factor model, since a latent variable (ability) and item characteristics (regression variables) are exogenous variables with respect to item performance (Y variable). Categorical or loglinear modeling is treated in the same way. The formal representation for the ANOVA approach (Grizzle, Starmer, & Koch, 1969) to categorical modeling is identical to ANOVA, with main effects and interactions to be represented. Logistic regression plays the analogous role to multiple regression with categorical outcome and interval predictors.

ESTIMATION AND STATISTICAL TESTING

Estimation of parameters in SEM is based on asymptotic multivariate normality and infinite sample size. A good deal of debate has occurred over the viability of normality assumptions for social science data, and I contend that this issue is not central to the use of SEM concepts. Using the Kruskal-Wallis test for an ANOVA design does not affect the conceptual nature of the SEM representation. Rank methods for contrasts exist if required. While the detail of the hypothesis tested, usually of medians rather than means,

may cause concern to some theorists, to the researcher the focus is on relationship between variables, and the hypothesis test is used as supporting evidence.

In estimation and hypothesis testing SEM has been a large sample size method. In recent years various simulations and Monte Carlo studies have been conducted to determine the lower limits of sample size for maximum likelihood procedures. For many applications, however, generalized or ordinary least squares methods are perfectly useful, providing a continuity between SEM and older statistical methods in GLM and measurement theory. There is every reason to expect further refinements and discoveries, as well as increasing use of bootstrap techniques and simulations, to provide reasonable evidence for specific estimation and hypothesis testing situations.

CONCLUSION

How researchers and teachers of social science research represent the fundamental methodology of their disciplines will have a major impact on how their students, and their students' students, conceive and conduct the next generations of research. What I have proposed here is intended to help organize and cohere that methodology.

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FIGURE CAPTIONS

Figure 1: Basic Two Variable Representations

Figure 2: Three Variable Representations

Figure 3: Factor Models

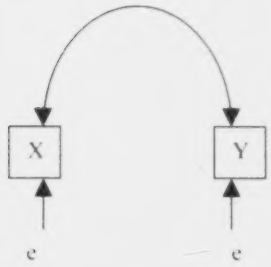
Figure 4: Four Variable Representations

Figure 5: Multivariate Representations

Figure 6: Nonlinear Model: Reciprocal Effects Model

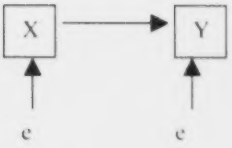
Figure 7: Nonlinear Model: Autoregressive (X) and moving average (Y) time-series cross-correlated processes

Figure 8: Taxonomy of Statistical Methods Based on SEM

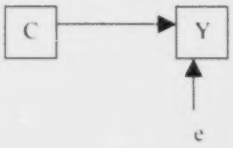


a. Two observed variable correlation

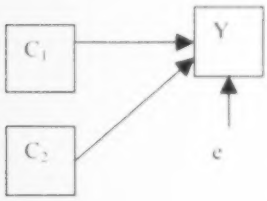
b. Directed two observed variable causal relationship



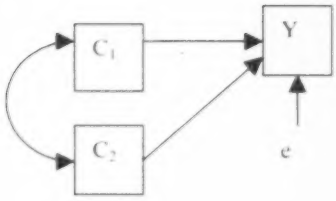
c. Regression model with fallible predictor



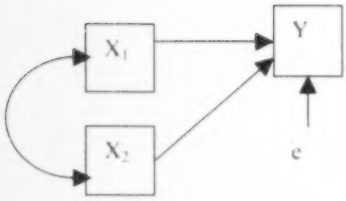
d. Experimental design model



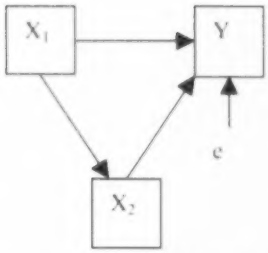
a. Two independent contrasts ANOVA



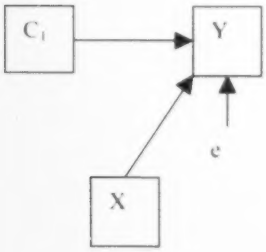
b. Two dependent contrasts ANOVA



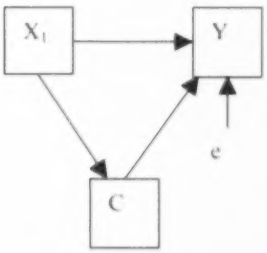
c. Classical multiple regression



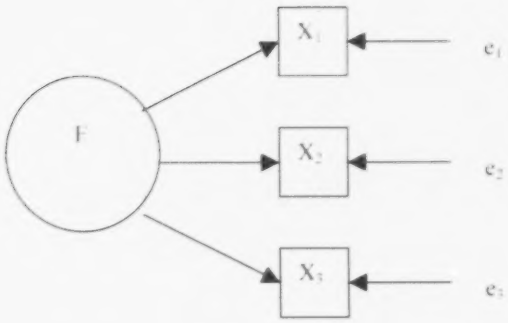
d. Classical path model with indirect effect



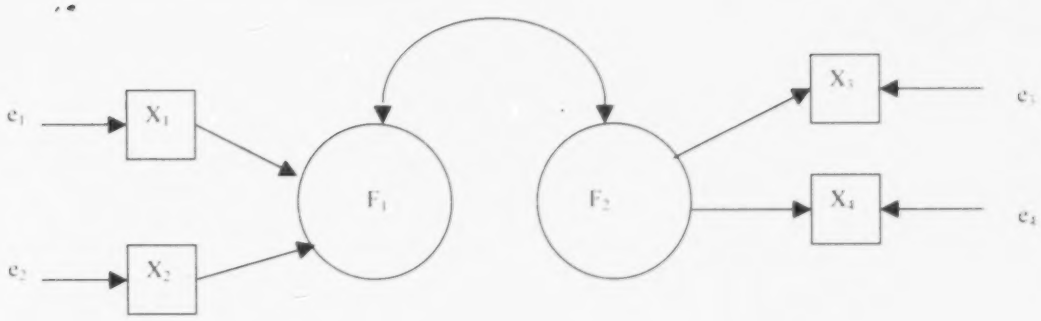
e. Classical randomized ANCOVA with 2 groups



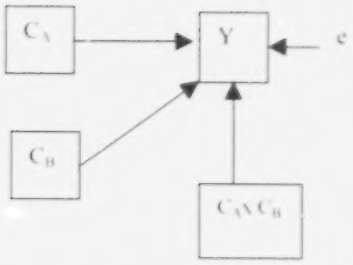
f. Nonrandomized ANCOVA



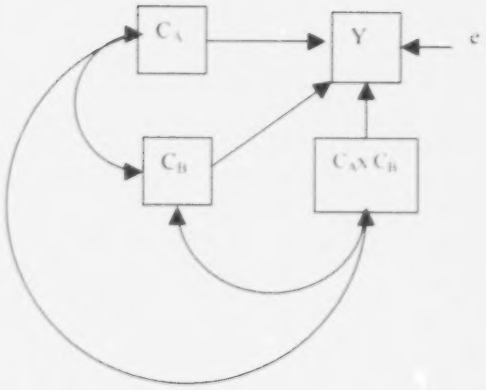
a. Classical minimal identified one factor structure



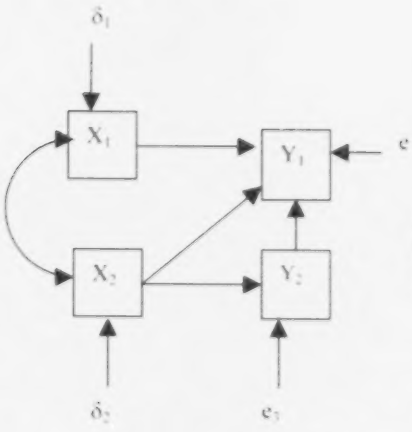
b. Minimally identifiable two-factor structure



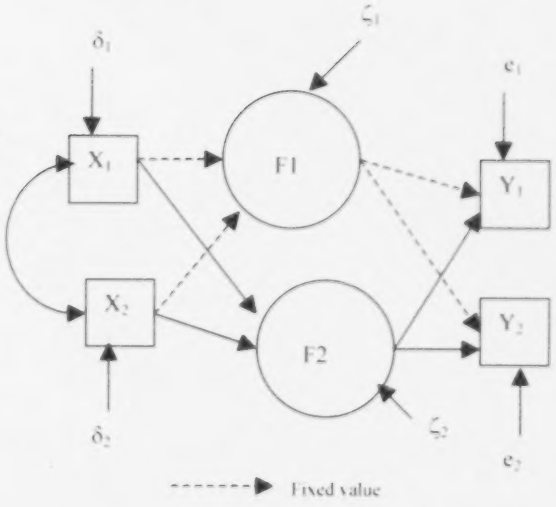
a. Two factor ANOVA balanced design with interaction



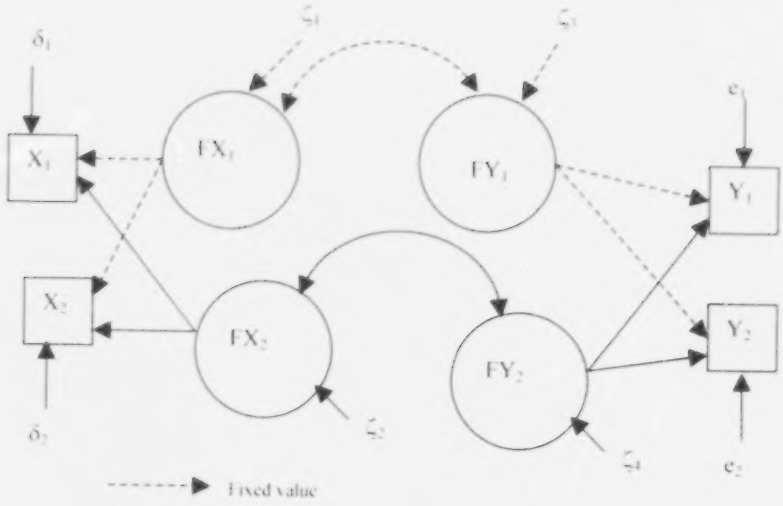
b. Two factor ANOVA unbalanced design with interaction



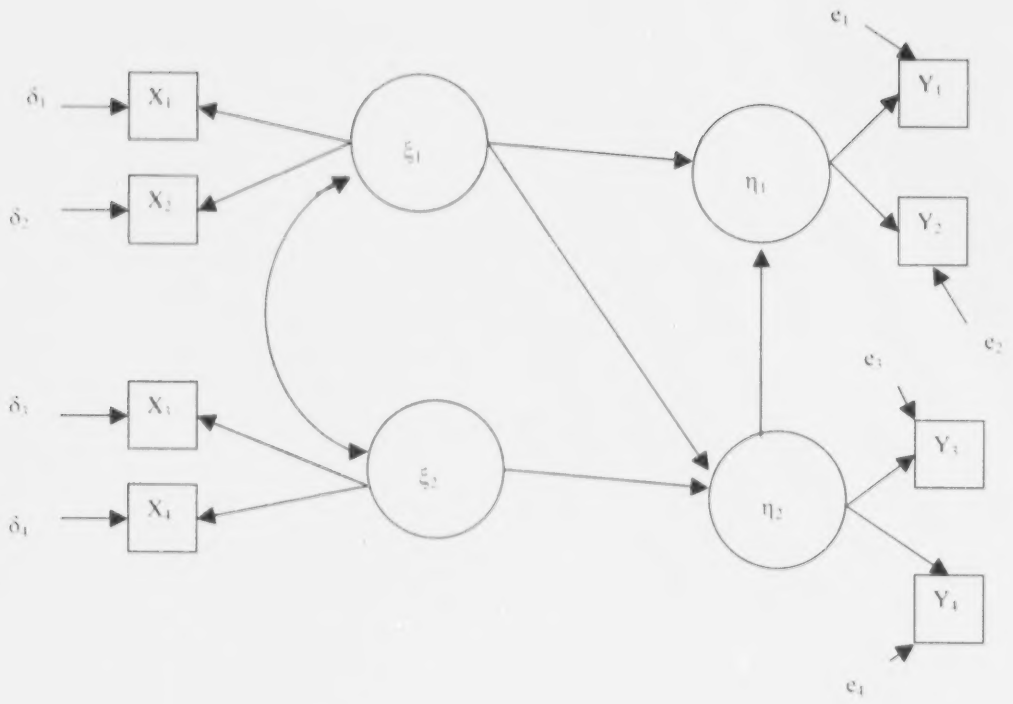
c. Path model with unanalyzed and indirect effects



d. Minimal canonical model



e: Theoretical representation of canonical correlation


δ1
δ2
δ3
δ4
X1
X2
X3
X4
ξ1
ξ2
η1
η2
e1
e2
e3
e4
Y1
Y2
Y3
Y4

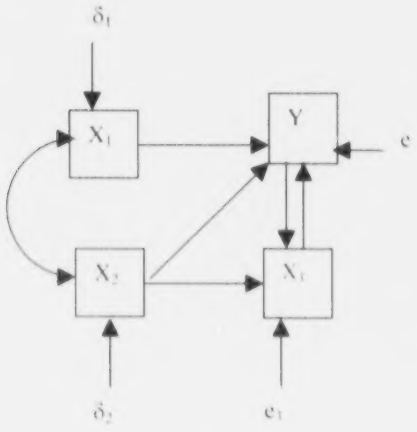
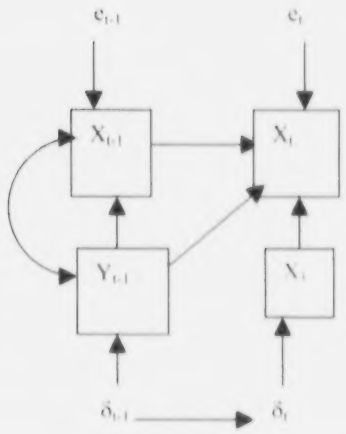


Fig. 6: Nonrecursive (nonlinear) path model



MULTIVARIATE STRUCTURAL MODELING